

Marwari college Darbhanga

Subject---physics (Hons)

Class--- B. Sc. Part 2

Paper—04 ; Group—A

Topic--- Growth of current in RL circuit. (Electricity)

Lecture series –52

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LR circuit

Growth and decay of current in L-R circuit

- Figure below shows a circuit containing resistance R and inductance L connected in series combination through a battery of constant emf E through a two way switch S

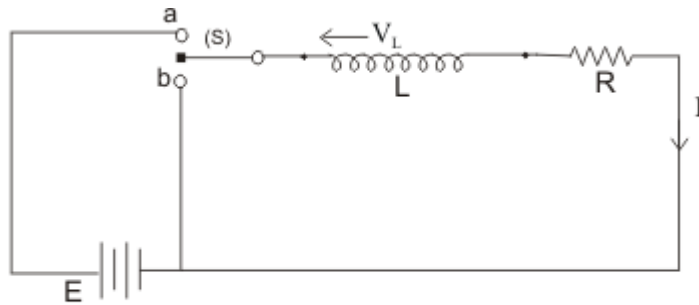


Figure 3. Circuit containing resistor and inductor

- To distinguish the effects of R and L , we consider the inductor in the circuit as resistance less and resistance R as non-inductive
- Current in the circuit increases when the key is pressed and decreases when it is thrown to b

(A) Growth of current

- Suppose in the beginning we close the switch in the up position as shown in below in the figure

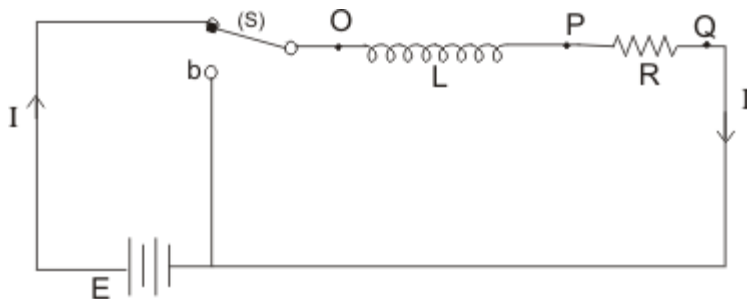


Figure 4. Battery is included in the circuit

- Switch is now closed and battery E , inductance L and resistance R are now connected in series
- Because of self induced emf current will not immediately reach its steady value but grows at a rate depending on inductance and resistance of the circuit

- Let at any instant I be the c current in the circuit increasing from 0 to a maximum value at a rate of increase dI/dt
- Now the potential difference across the inductor is $V_{op}=LdI/dt$
and across resistor is

$$V_{pq}=IR$$

Since

$$V=V_{op}+V_{pq}$$

so we have,

$$V = L \frac{dI}{dt} + IR \quad \text{---(6)}$$

Thus rate of increase of current would be,

$$\frac{dI}{dt} = \frac{V - IR}{L} \quad \text{---(7)}$$

- In the beginning at $t=0$ when circuit is first closed current begins to grow at a rate,

$$\left(\frac{dI}{dt} \right)_{t=0} = \frac{V}{L}$$

from the above relation we conclude that greater would be the inductance of the inductor, more slowly the current starts to increase.

- When the current reaches its steady state value I , the rate of increase of current becomes zero then from equation (7) we have,

$$0=(V-IR)/L$$

or,

$$I=V/R$$

From this we conclude that final steady state current in the circuit does not depend on self inductance rather it is same as it would be if only resistance is connected to the source

- Now we will obtain the relation of current as a function of time Again consider equation (6) and rearrange it so that

$$\frac{dI}{\left(\frac{V}{R}\right) - I} = \frac{R}{L} dt$$

let $V/R = I_{\max}$, the maximum current in the circuit .so we have

$$\frac{dI}{I_{\max} - I} = \frac{R}{L} dt \quad \text{---(8)}$$

- Integrating equation (8) on both sides we have

$$-\ln(I_{\max} - I) = \frac{R}{L} t + C \quad \text{---(9)}$$

where C is a constant and is evaluated by the value for current at $t=0$ which is $i=0$

so,

$$C = -\ln(V/R) = -\ln I_{\max}$$

putting this value of C in equation (9) we get,

$$\ln\left(\frac{I_{\max} - I}{I_{\max}}\right) = -\frac{R}{L} t$$

Or,

$$\frac{I_{\max} - I}{I_{\max}} = e^{-\frac{R}{L} t}$$

Or,

$$I = I_{\max} (1 - e^{-\frac{R}{L} t}) \quad \text{---(10)}$$

This equation shows the exponential increase of current in the circuit with the passage of time

- Figure below shows the plot of current versus time

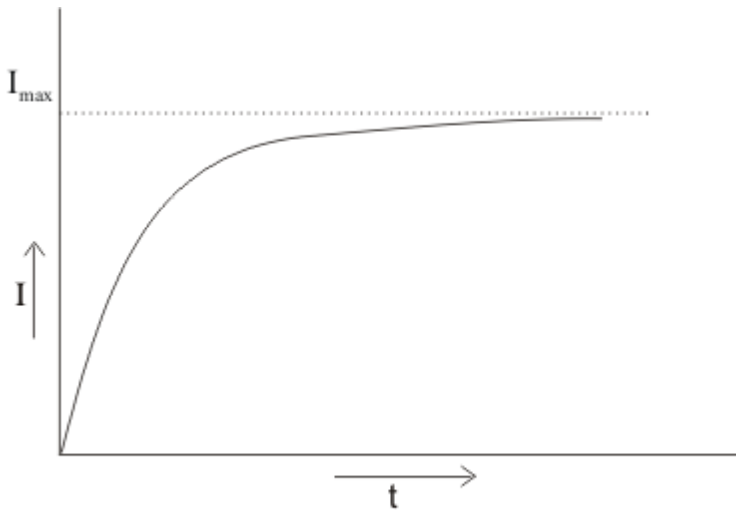


Figure 5. Growth of current in circuit containing inductance and resistance

- If we put $t = \tau_L = L/R$ in equation 10 then,

$$I = I_{\max} \left(1 - \frac{1}{e}\right) = .63 I_{\max}$$

Hence, the time in which the current in the circuit increases from zero to 63% of the maximum value of $I_{\max}$ is called the constant or the decay constant of the circuit.

- For LR circuit, decay constant is,
 $\tau_L = L/R$ ---(11)

- Again from equation (8),

$$\frac{dI}{dt} = \frac{R}{L} (I_{\max} - I_0) = \frac{I_{\max} - I_0}{\tau_L}$$

Or,

$$\frac{dI}{dt} \propto \frac{1}{\tau_L}$$

This suggests that rate of change current per sec depends on time constant.

- Higher is the value of decay constant, lower will be the rate of change of current and vice versa.

